TEM Pseudoelliptic-Function Bandstop Filters Using Noncommensurate Lines

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Abstract—**A synthesis procedure for distributed bandstop filters that approximate an elliptic-function response is described. Existing tables of lumped constant elliptic-function filters are used together with an approximate transform that exchanges commensurate Foster sections for noncommensurate pairs of stubs or coupled lines. The method is usable for all bandwidths up to about 100 percent, providing that networks with a relative bandwidth of more than 50 percent are considered to be pseudo-low-pass.**

Introduction

THE advantages of elliptic-function filters with respect to high rates of cutoff are well known and several authors have presented design procedures for such filters when applied to TEM distributed parameter networks. Horton and Wenzel [1] proposed a method based on nonredundant network synthesis, but found the procedure tedious and the networks nonrealizable in most cases, due to the difficulty of the physical constructions associated with microwave Brune sections. The use of digital lines in a design based on a lumped-element prototype [2], [3] has found wide application but is inherently wide band, and can only be constructed in machined form.

The methods proposed by Levy and Whiteley [4] and Schiffman and Young [5] are based on the use of redundant unit elements, employing the transforms of Kuroda [6] and Kuroda–Levy [7]. These methods are the only ones suitable for the synthesis of etched (stripline or microstrip) bandstop filters with an elliptic-function response. In all cases limits of physical realizability are set by the inability to construct the necessary Foster or Brune sections, leaving a gap in realizable bandwidth between approximately 10 and 80 percent.

Common to all the above mentioned design methods is the use of Richards' [8] transform. Under this transform open- and short-circuit stubs of the *same* (commensurate) length are transformed to capacitors (C) and inductors (L). In specifying that all elements must be commensurate, ease of synthesis (and analysis) is thus gained, at the expense of one order of freedom. Consequently, it is in most cases necessary to introduce redundancy by means of unit elements (quarter-wavelength lines) which have no direct lumped-element equivalent under Richards' transform but are extremely useful in TEM network theory and in this instance serve to ease realizability. Unit elements are either introduced in a nonredundant synthesis procedure, in which case they contribute to the network performance, or through redundant transformations from the input and output ports in which case they leave the amplitude response of the network unaltered.

Fig. 1 shows the effect of Richards' transform on a lumped-element fifth-order elliptic-function low-pass filter.

Transmission zeros occur at the center (commensurate) frequency of the distributed response, as well as in pairs equispaced about ω_0. Each distributed Foster or Brune section contributes one pair of zeros, while remaining commensurate.

In this paper a method is proposed in which TEM Foster sections are exchanged for pairs of *noncommensurate* stubs[1] with their transmission zeros equispaced about the center frequency ω_0 that yields a design that has a pseudo-elliptic-function bandstop response.

Noncommensurate Parallel-Foster Approximation

Consider an open circuit and a short-circuited stub of electrical length θ, under the transform

$$p = j \tan \theta/2.$$

The input impedances then become, respectively,

$$Z_{SO} = pL + \frac{1}{pC}, \qquad L = Z_0/2, \qquad C = 2/Z_0$$

and

$$Z_{SC} = \frac{pL\left(\frac{1}{pC}\right)}{pL + \frac{1}{pC}}, \qquad L = 2Z_0, \qquad C = \frac{1}{2Z_0}.$$

By application of this transform to the network in Fig. 2(a), which is a parallel-Foster section as shown in Fig. 2(b) under Richards' transform, the equivalent network of Fig. 2(c) is obtained.

Consider the circuits of Fig. 2(c) and (d). By equating the admittances of the parallel networks, it is found that they are identical if

$$Z_A = \frac{Z_1}{2}(1 + 1/f^2) = Z_D$$

$$Z_B = \frac{Z_1}{2}(1 + f^2) = Z_C$$

[1] For simplicity the word "stub" in this context is taken to include parallel coupled lines, etc., that yield a cascade of unit element and TEM capacitor.

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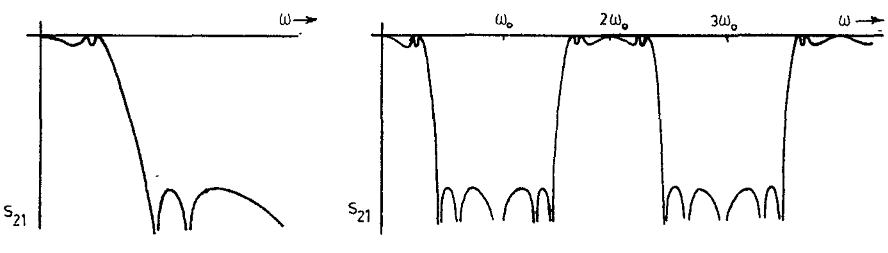
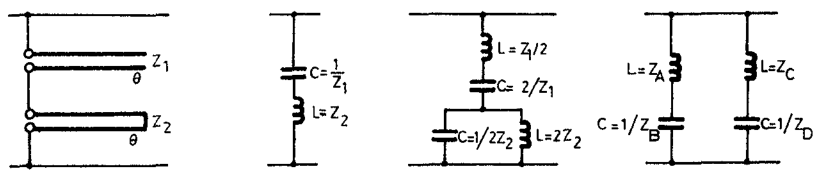


(a) (b)

Fig. 1. Elliptic-function filter response. (a) Lumped-element low pass. (b) Distributed bandpass.



(a) (b) (c) (d)

Fig. 2. (a) Equivalent representations of microwave Foster sections. (b) Under Richards' transform. (c) Under the p-transform. (d) An equivalent of (c).

with

$$f = \sqrt{k+1} + \sqrt{k} \tag{1}$$

$$\sqrt{k} = Z_2/Z_1. \tag{2}$$

The frequencies $f = \sqrt{Z_B/Z_A}$ and $1/f$ are the resonant frequencies of the two parallel-Foster circuits of the network of Fig. 2(d). The admittance of this circuit can be written as

$$Y_c(p) = \frac{1}{Z_0(p/f + f/p)} + \frac{1}{Z_0(fp + 1/fp)} \tag{3}$$

where

$$Z_0 = \frac{Z_1}{2}(f + 1/f).$$

Clearly, (3) describes the parallel connection of two open-circuit stubs of the same characteristic impedance $2Z_0$ scaled to resonate at frequencies of f and $1/f$, respectively. A noncommensurate approximation to this network is now defined as two parallel stubs, resonating at f and $1/f$, with their impedances adjusted to give the same admittance poles and zeros as $Y_c(p)$, up to a frequency just below $2\omega_0$. Defining

$$\omega_1 = \omega_0 \frac{4}{\pi} \tan^{-1} f$$

$$\omega_2 = \omega_0 \frac{4}{\pi} \tan^{-1} 1/f$$

$$p_1 = j \tan\left(\frac{\pi}{4}\frac{\omega}{\omega_1}\right)$$

$$p_2 = j \tan\left(\frac{\pi}{4}\frac{\omega}{\omega_2}\right)$$

then

$$y_n(p_1,p_2) = \frac{1}{Z_0 f'(p_1 + 1/p_1)} + \frac{1}{Z_0/f'(p_2 + 1/p_2)}.$$

To obtain the same position for the admittance pole of $y_c(p)$ and $y_n(p_1,p_2)$, the following must hold:

$$[f'(p_1 + 1/p_1) + 1/f'(p_2 + 1/p_2)]|_{\omega=\omega_0} = 0.$$

Setting

$$\theta_1' = \left(\frac{\pi}{4}\right)^2 \frac{1}{\tan^{-1} f}$$

$$\theta_2' = \left(\frac{\pi}{4}\right)^2 \frac{1}{\tan^{-1} 1/f} \tag{4}$$

it follows that

$$f' = \sqrt{\frac{\cot\theta_2' - \tan\theta_2'}{\tan\theta_1' - \cot\theta_1'}}. \tag{5}$$

The characteristic impedances are then given by

$$Z_1' = 2Z_0 f' \qquad Z_2' = 2Z_0/f' \tag{6}$$

and the stub lengths by

$$l_1 = \frac{v}{8\pi\omega_1} \qquad l_2 = \frac{v}{8\pi\omega_2} \tag{7}$$

where v is the propagation velocity in the dielectric.

The noncommensurate approximation improves as the transmission zeros of the prototype filter approach the center frequency of ω_0, and degenerates into a commensurate network if all transmission zeros lie at ω_0. The transmission zeros move away from ω_0 with increasing bandwidth as well as increasing modular angle of the elliptic-function prototype, causing an increase in error due to the non-

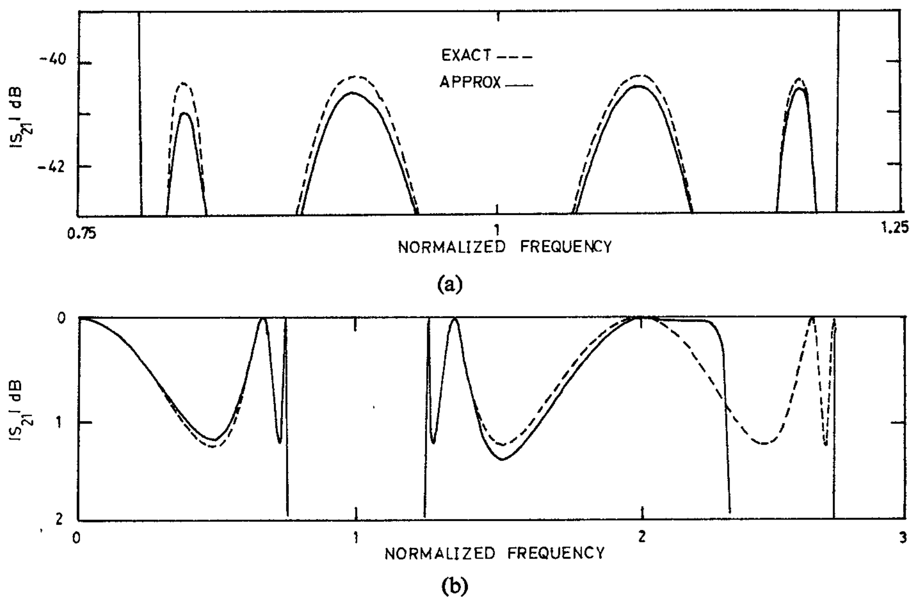


Fig. 3. Calculated transmission response for fifth-order bandstop filter, for both exact and approximate realizations. (a) Expanded response of the first stopband. (b) Expanded response of the passbands.

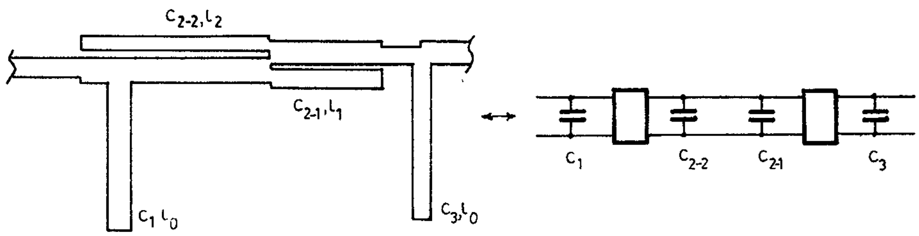


Fig. 4. Physical noncommensurate realization.

TABLE I
PERFORMANCE COMPARISON BETWEEN EXACT VALUES AND APPROXIMATION (dB)

ρ%	MODULAR ANGLE (θ)	BANDWIDTH %	LOWER PASSBAND		STOP BAND		UPPER PASSBAND	
			EXACT	APPROX.	EXACT	APPROX.	EXACT	APPROX.
50	56°	20	1,25	1,23	40,3	40,4	1,25	1,25
		30		1,21		40,5		1,25
		50		1,18		40,9		1,37
10	50	30	0,045	0,041	31,02	31,16	0,045	0,047
		50		0,035		31,52		0,062

commensurate approximation. Fig. 3 shows the calculated transmission response of a fifth-order elliptic-function bandstop filter with a modular angle of 56°, a 50-percent (−1.25-dB) ripple and a bandwidth of 50 percent, for both the approximation and the exact Foster sections.

Table I shows calculated error values of two prototypes for various bandwidths. In all cases the approximation is seen to have slightly less attenuation in the lower passband, slightly more in the upper passband, and an increased stopband attenuation.

SYNTHESIS PROCEDURE

Due to the extremely wide range of bandwidths that can be achieved by this type of filter, a simple unified synthesis procedure is not possible. A general approach can, however, be outlined and is illustrated by two design examples. Firstly, a prototype that will give the desired response is chosen from tables [9]. The prototype is then scaled for bandwidth and impedance [6]. As realization is simplest when the final network consists of shunt capacitors spaced by unit elements, a T-type network must be chosen for third-order designs, a Π-type for fifth order, and so on.

By means of Kuroda and Kuroda–Levy transforms, unit elements are now transformed into the network from the input and output ports, until a realizable network is obtained; this is indicated when the network consists only of alternate parallel capacitors and shunt Foster sections spaced by unit elements. A good description of this process is given in [5].

The next step is to replace each shunt Foster by its noncommensurate pair of shunt "capacitors" (stubs). The impedance of each stub is then evaluated; if the impedance is below about 150 Ω,[2] it can be realized as a shunt stub. Between about 150 and 300 Ω, Schiffman's spurline [10] is used and above 300 Ω, realization is by means of coupled lines. Sections of the type shown in Fig. 4 can be used to realize the noncommensurate elements. A proof of this type of network is given in the Appendix.

[2] An exact value cannot be given here as this would vary with the material used. Quoted values are for the materials mentioned in the design examples.

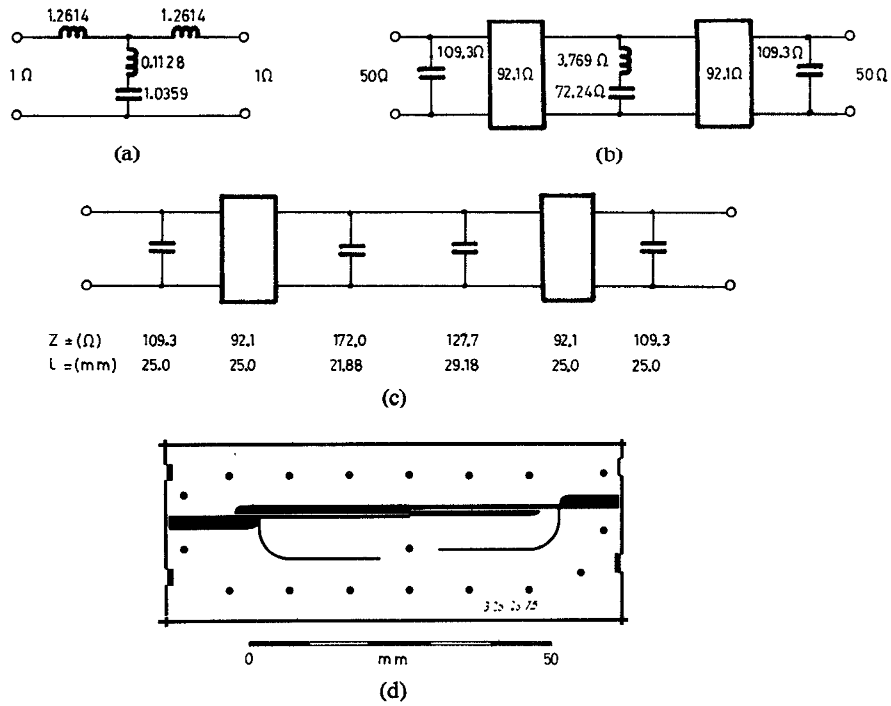


Fig. 5. Design of a third-order bandstop filter. (a) Lumped-element low-pass elliptic-function prototype, unscaled. (b) Impedance and bandwidth scaled and transformed network. (c) Its noncommensurate approximation. (d) Etching mask.

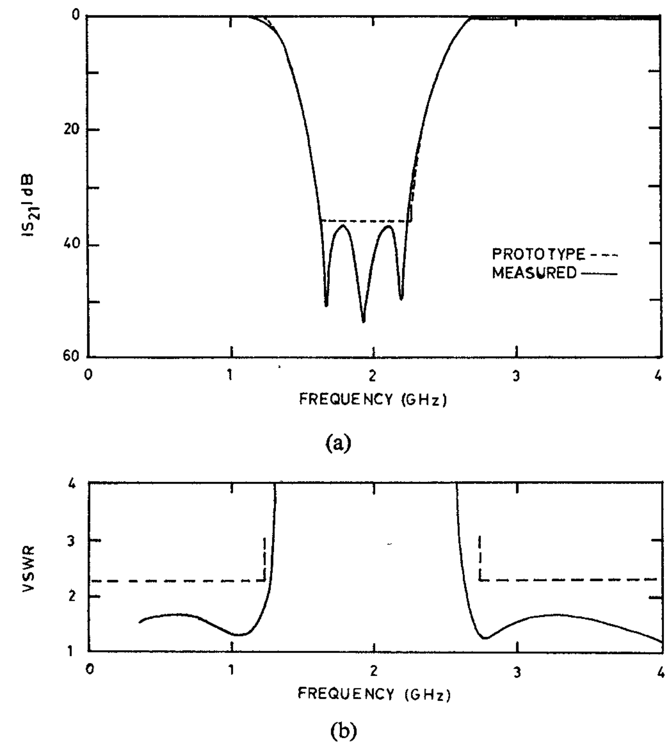


Fig. 6. Frequency response of the filter of Fig. 5. (Third order.)

Design Examples

A. Third-Order Network

Although very simple, an example of a third-order network is included because it illustrates the principle involved rather clearly. Fig. 5(a) shows a third-order lumped-element prototype with a -0.28-dB (25-percent) ripple level and modular angle $\theta = 23°$. After impedance (50 Ω) and bandwidth (75-percent) scaling and the application of Kuroda transforms over the end inductors, the network of Fig. 5(b) is obtained. Application of the noncommensurate approximation yields the configuration of Fig. 5(c): line lengths and impedances are shown for a 1.97-GHz center frequency.

The filter was constructed in 3.175-mm ($\frac{1}{8}$-in) GPS polyolefin stripline with the end sections realized as stubs, folded back to conserve space. The noncommensurate stubs are realized as spurlines. The physical layout of the etching mask is shown in Fig. 5(d). The measured frequency response characteristics are shown in Fig. 6.

B. Fifth-Order Network

As a fifth-order design example, a prototype with -0.18-dB (20-percent) ripple level and 40° modular angle was chosen [Fig. 7(a)]. The bandwidth is 50 percent and the filter resonates at 1.97 GHz. After the necessary scaling and transforms, the network of Fig. 7(b) is obtained.

All sections except the 145-Ω stub C_{2-2} have been manufactured as spurlines. The center pattern is shown in Fig. 7(c). Construction was in 6.35-mm ($\frac{1}{4}$-in) GPS polyolefin. Frequency response characteristics are shown in Fig. 8.

Conclusions

A design procedure that yields easily manufactured etched stripline bandstop filters that approximate an elliptic-function filter response, has been presented. The proposed method can be used with accuracy up to 50-percent bandwidth, providing the form of the spurious responses is not of importance, and up to 80 or 100 percent in bandwidth if the network is to be used in low-pass applications.

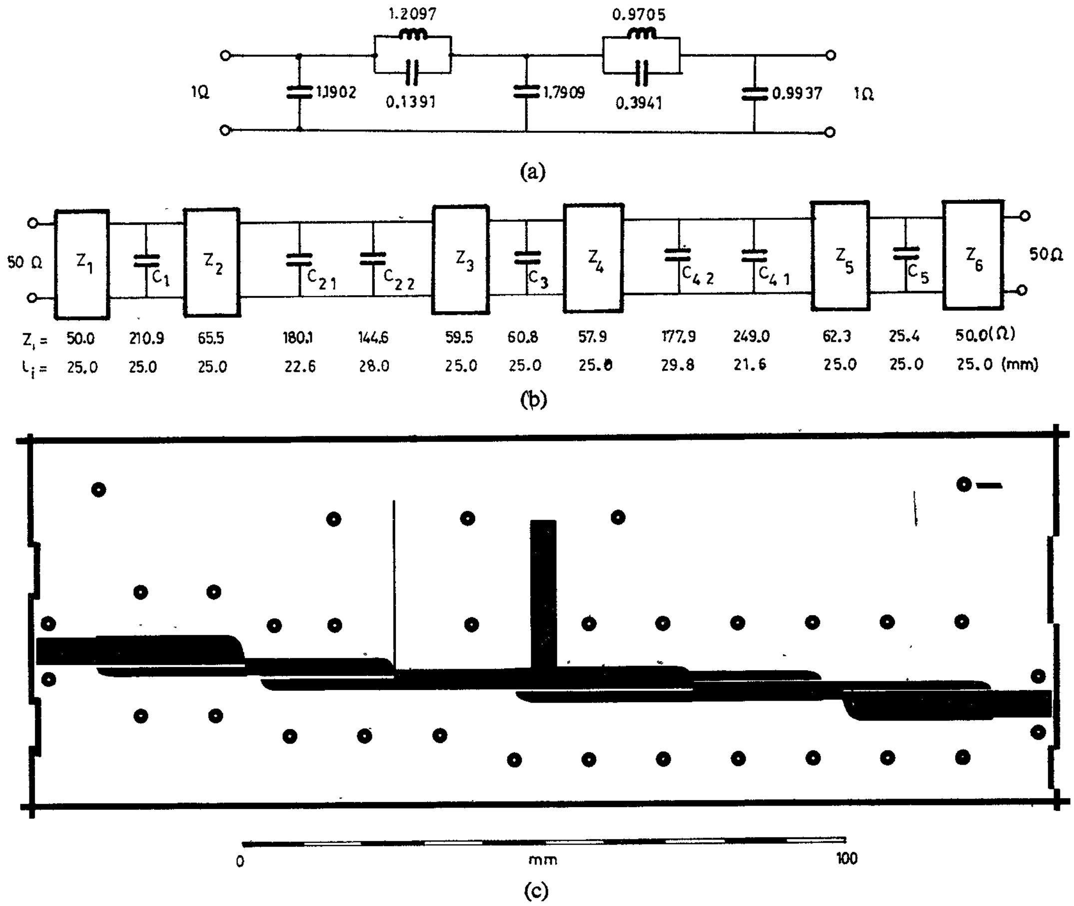


Fig. 7. Fifth-order filter design. (a) Lumped-element prototype. (b) Scaled and transformed network showing line lengths for 1.97-GHz center frequency. (c) Etching mask.

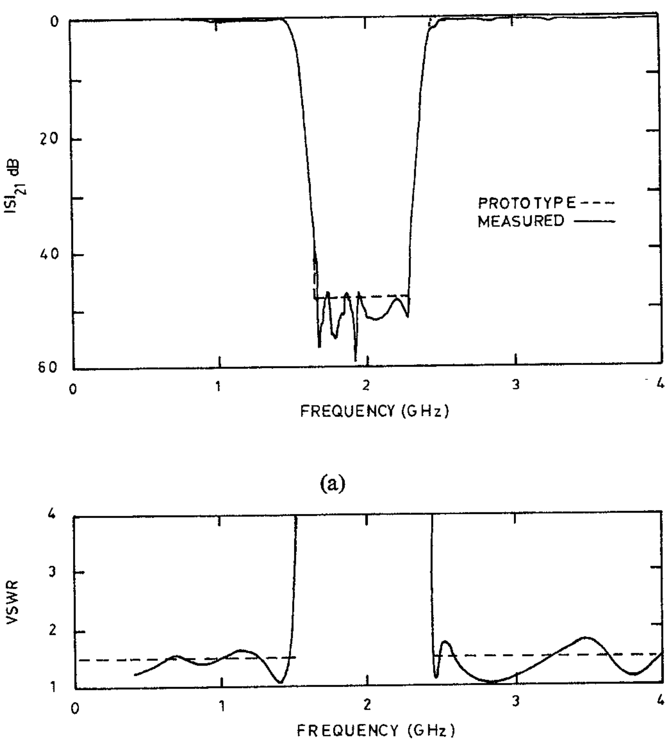


Fig. 8. Frequency response of fifth-order bandstop filter.

Appendix

An equivalent circuit for two coupled lines is derived by Wenzel [11] by transforming the capacitance matrix, as shown in Fig. 9(a) and (b). The capacitance matrix of Fig. 9(a) is given by

$$[c] = \begin{bmatrix} c_{11} + c_{12} & -c_{12} \\ -c_{12} & c_{12} + c_{22} \end{bmatrix}.$$

By multiplying the last row and column by n', then

$$[c]' = \begin{bmatrix} c_{11} + c_{12} & -n'c_{12} \\ -n'c_{12} & n'^2(c_{12} + c_{22}) \end{bmatrix}.$$

If

$$n' = \frac{c_{12}}{c_{12} + c_{22}}$$

then $c_{22}' = 0$ and the network of Fig. 9(b) results. The equivalent of this configuration is given by the network of Fig. 9(c): between 1-1′ and 2-2′ there exists a unit element, of characteristic impedance

$$Z_1 = \frac{\eta}{\sqrt{\varepsilon_r}\, c_{11}'/\varepsilon}$$

with a unit element

$$Z_2 = \frac{\eta}{\sqrt{\varepsilon_r}\, c_{12}'/\varepsilon}$$

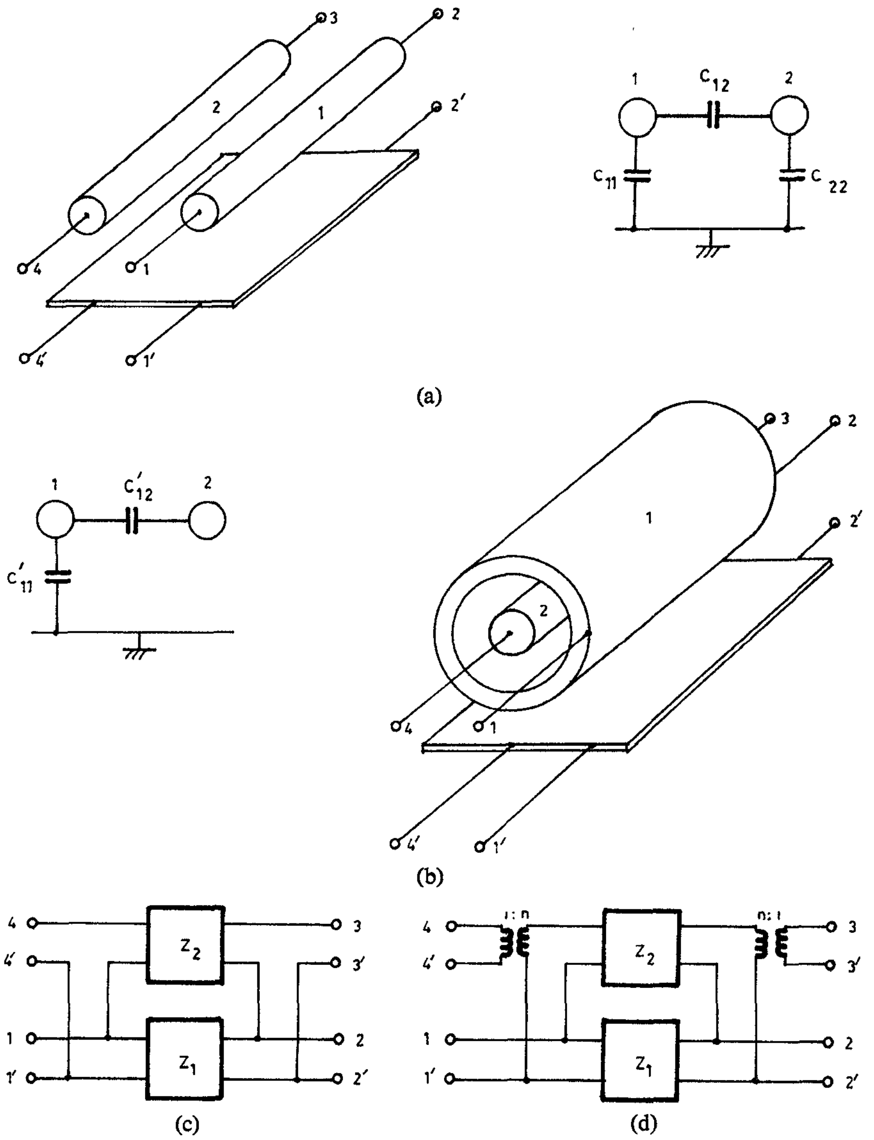


Fig. 9. Capacitance matrix transformation for the derivation of a network model for coupled lines, (a). The capacitance matrix is transformed, (b), with the equivalent circuit as in (c). By transforming back, the network model (d) is obtained.

connected between 4-1 and 3-2, 4′ and 1′, and 3′ and 2′ being the same points. As Wenzel points out, this is equivalent to connecting a transformer of turns ratio $1/n'$ in cascade with 4-4′ and 3-3′.

If, instead of applying the relevant port conditions, the transformers, of turns ratio $n = 1/n'$ are inserted, the four-port network model of Fig. 9(d) results, with

$$n = 1 + \frac{c_{22}}{c_{12}}.$$

With known values of Z_1, Z_2, and n, the normalized capacitances can then be calculated from

$$c_{11}/\varepsilon = \frac{\eta}{\sqrt{\varepsilon_r}}\left\{\frac{1}{Z_1} - (n-1)\frac{1}{Z_2}\right\}$$

$$c_{12}/\varepsilon = \frac{\eta}{\sqrt{\varepsilon_r}}\frac{n}{Z_2}$$

$$c_{22}/\varepsilon = \frac{\eta}{\sqrt{\varepsilon_r}}\frac{n(n-1)}{Z_2}.$$

The relationship between c_{11}, c_{12}, c_{22} and line dimensions is well known [12].

Consider next the network as shown in Fig. 10(a), with the equivalent model of Fig. 10(b), in which line lengths and impedances are indicated. Providing the transformer turns n: 1 remain the same, the transformers (A) can be removed, to yield the network of Fig. 10(c) which simplifies to 10(d) and proves the network equivalence. This method is readily linearly extended to include three coupled lines and consequently makes the analysis of very complicated networks feasible.

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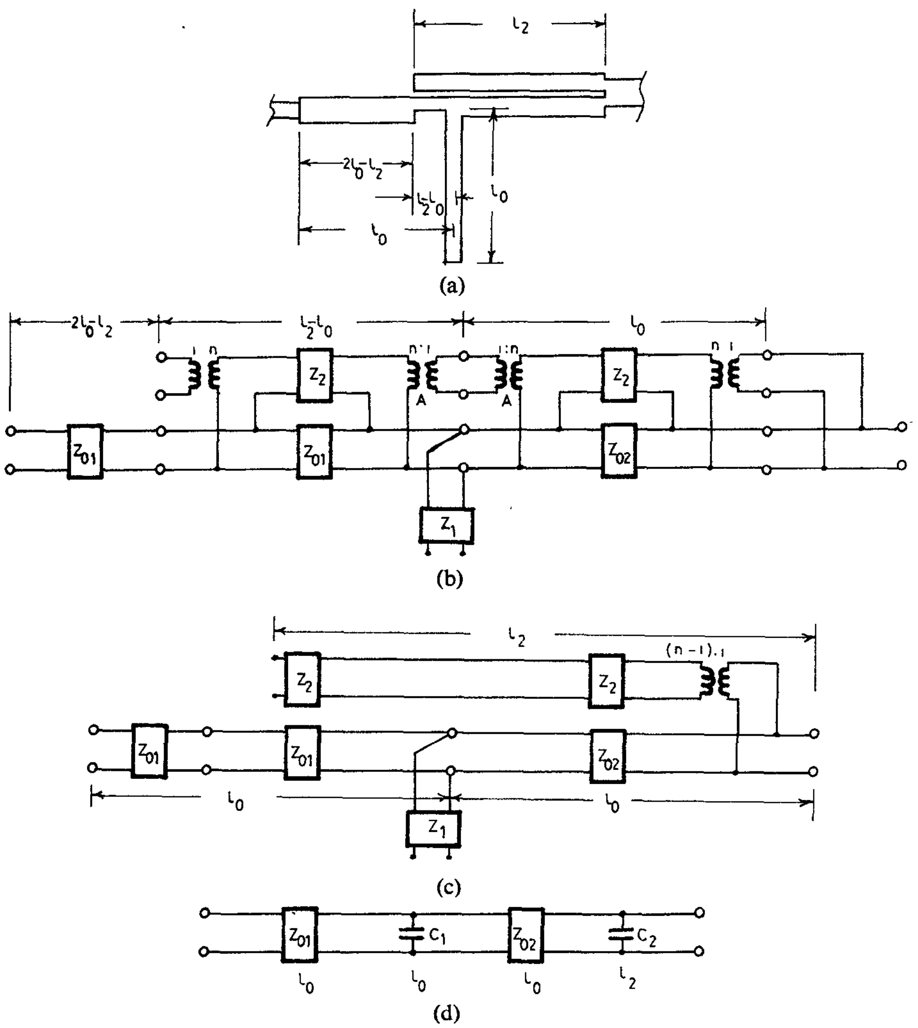

Fig. 10. Proof of the network form for noncommensurate lines.

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